

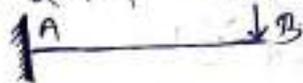
Ch-3 Bending moments and Shearing force

① Point load :- एक बिंदु लोड या केन्द्रित भार वह होता है जिसे पौंट के मे काम करने के लिए माना जाता है। वास्तविक अवस्था में, भार के एक छोटे से क्षेत्र में वितरित किया जाना चाहिए क्योंकि इस तरह तरह के छोटे चार्ज - किनारे के संपर्क आमतौर पर न ही संभव है और न ही वाहनीय।

② Distributed load :- वितरित भार वह होता है जिसे किसी body के सतह पर काम करने के लिए माना जाता है। वास्तविक में भार किसी सतह पर वितरित होता है।

Types of beam

① Cantilever beam :- इस बिम का एक सपोर्ट फिक्स रहता है तथा दूसरा सपोर्ट फ्री रहता है जिसमें लोड लागू होता है।



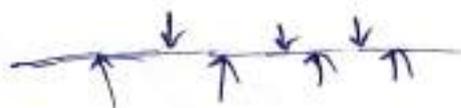
② Simply supported beam :- इस बिम के दो अंतिम सपोर्टों में दो सपोर्ट हैं लेकिन उनके बीच में लगातार लोड और सपोर्ट के लिए रहता है।



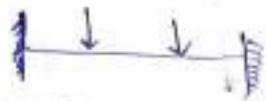
③ Continuous beam :- इस बिम में दो सपोर्टों के लिए होते हैं और उसके बाएँ एवं दाएँ में सपोर्टों के लिए रहता है।



④ Overhanging beam :- एक ओवरहैंगिंग बीम वह है जिसमें सपोर्टों का संपर्क नहीं होता है यानी कि एक या दो सपोर्टों के सपोर्टों से परे है इसके सपोर्टों से बिम आगे निकल जाता है।

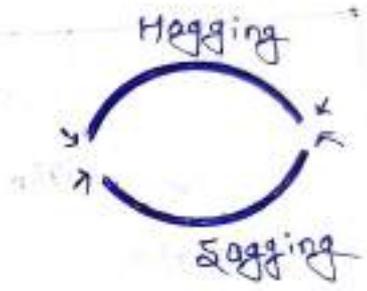
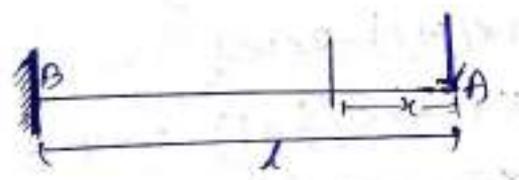


③ Fixed beam :- एक निश्चित बॉम वह है जिसका दोनों छंत छोरे रूप से तय या इसके अंदर सहायक दीवारों या स्तंभों में होता है।



Shear Force :- इसी तरह 'नकारात्मक' कुतन बल। एक कुतनी बल दाहिने छाप की ओर एक छप की दिशा में धारा या कप की दिशा अनुभाग के कई ओर या नीचे की दिशा में नीचे की दिशा में नकारात्मक लिया जाएगा। इसी तरह 'नकारात्मक' कुतन बल ~~अपने डा एरि एक~~ इसमें Right hand side में हल कला है। $\downarrow (-) \uparrow (+)$

Bending moment :-



S.F. at $x = S_x = -w$
 B.M. at $x = M_x = -wx$

$x = 0$
 $B.M = 0$
 $x = l$
 $B.M = wl$



CHAPTER - 4 Bending Stresses

Imp

Bending eqn. \rightarrow

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

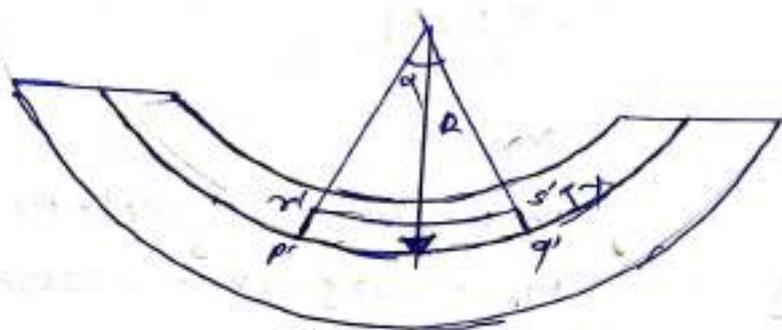


Before bending $r_s = r_p$

After Bending

$$r_s = r'_s$$

$$p_q = p'_q$$



Strain = $\frac{\text{Change in length}}{\text{original length}}$

$$r'_s = (R - y)^2, \quad p'_q = R^2$$

$$r_s = p_q = p'_q = R^2$$

$$\text{In } r_s = \frac{r_s - r'_s}{r_s}$$

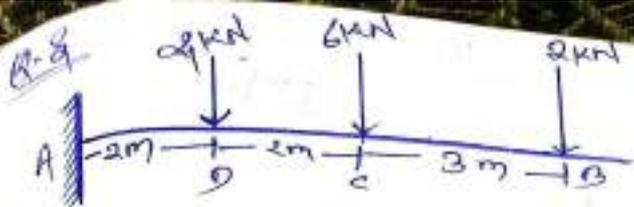
$$= \frac{R^2 - (R - y)^2}{R^2}$$

$$= \frac{R^2 - R^2 + 2Ry + y^2}{R^2}$$

$$\approx \frac{2Ry}{R^2} = \frac{y}{R}$$

$$\frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R} \quad \text{--- (A)}$$

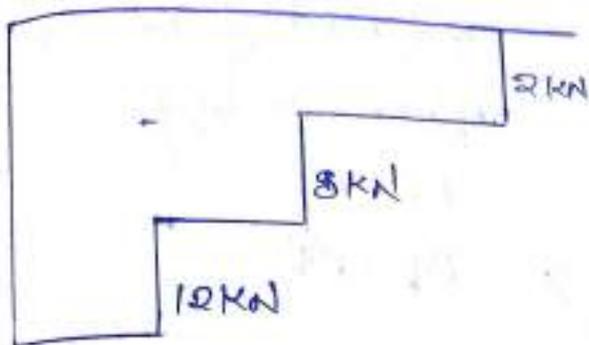


S.F

$$S_B = -2 \text{ kN}$$

$$S_C = -2 - 6 = -8 \text{ kN}$$

$$S_D = -2 - 6 - 4 = -12 \text{ kN}$$



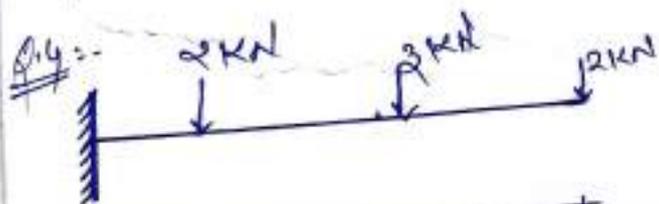
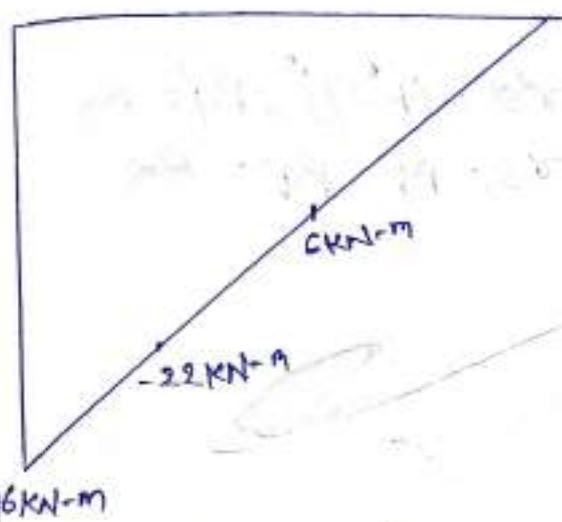
B.M

$$M_B = 0$$

$$M_C = -2 \times 3 = -6 \text{ kN-m}$$

$$M_D = -2 \times 5 - 6 \times 2 = -22 \text{ kN-m}$$

$$M_A = -2 \times 7 - 6 \times 4 - 4 \times 2 = -46 \text{ kN-m}$$

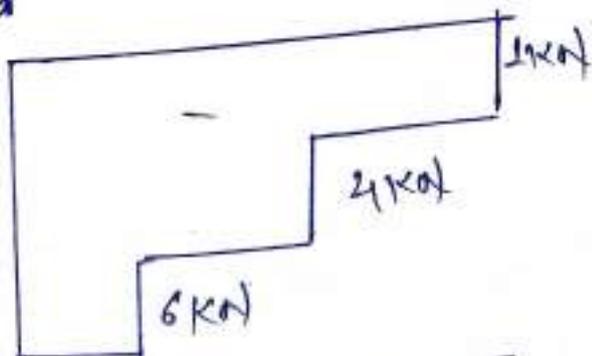


S.F

$$S_B = -1 \text{ kN}$$

$$S_C = -1 - 2 = -3 \text{ kN}$$

$$S_D = -1 - 2 - 2 = -5 \text{ kN}$$



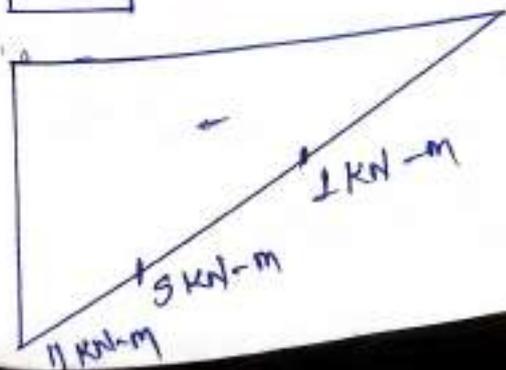
B.M

$$M_B = 0$$

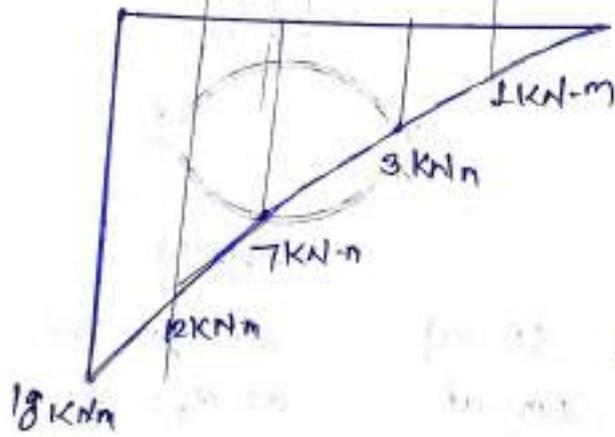
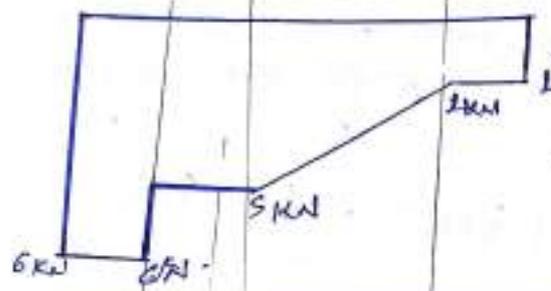
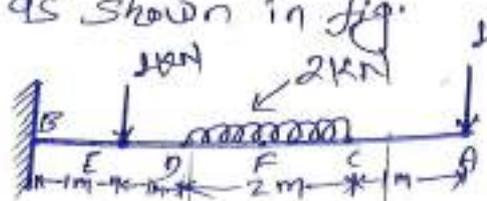
$$M_C = -1 \times 1 = -1 \text{ kN-m}$$

$$M_D = -1 \times 2 - 2 \times 1 = -5 \text{ kN-m}$$

$$M_A = -1 \times 3 - 2 \times 2 - 2 \times 1 = -11 \text{ kN-m}$$



Q → Draw the S.F. and B.M diagrams for beam as shown in fig.



SF

$$S_A = -1 \text{ kN}$$

$$S_D = -1 - 2 \times 2 = -5 \text{ kN}$$

$$S_E = -5 - 1 = -6 \text{ kN}$$

$$S_B = -6 \text{ kN}$$

B.M

$$M_A = 0$$

$$M_C = -1 \times 1 = -1 \text{ kNm}$$

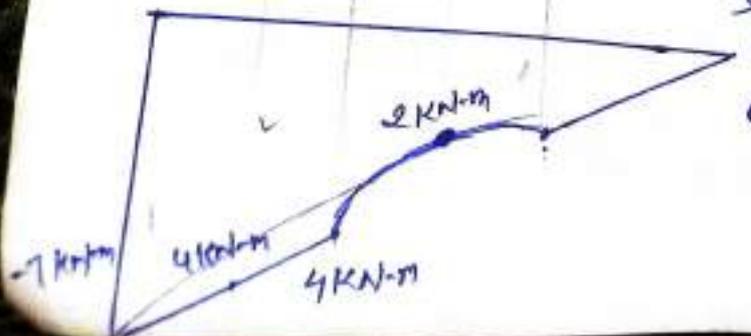
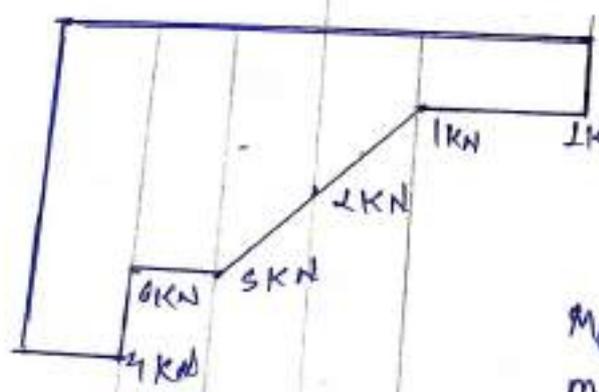
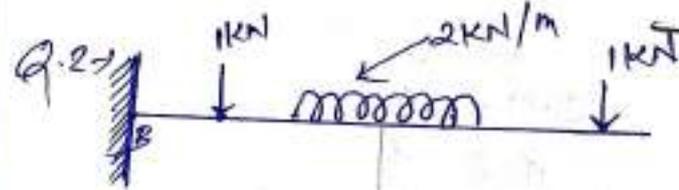
$$M_D = -1 \times (1+2) - 2 \times 2 \times \frac{2}{2} = -3 - 4 = -7$$

$$M_E = -1 \times (4+1) - 2 \times 1 \times \frac{1}{2} = -2 - 1 = -3 \text{ kNm}$$

$$M_F = -1 \times 4 - 2 \times \left(\frac{2}{2} + 1\right) = -4 - 8 = -12 \text{ kNm}$$

$$M_G = -1 \times 5 - 2 \times \left(\frac{2}{2} + 1 + 1\right) - 1 \times 1$$

$$= -5 - 12 - 1 = -18 \text{ kNm}$$



S.A) $S_A = 1 \text{ kN}$

$$S_E = -2 \times 0 = 0 \text{ kN}$$

$$S_F = -2 \times 1 = -2 \text{ kN}$$

$$S_D = -2 \times 2 = -4 \text{ kN}$$

$$S_E = 1 \times 1 = 1 \text{ kN}$$

$$S_B = 1 - 2 - 1 = -2 \text{ kN}$$

B.M

$$M_A = 0, M_C = 0$$

$$M_E = -2 \times 1 \times \frac{1}{2} = -1 \text{ kNm}$$

$$M_D = -2 \times 2 \times \frac{2}{2} = -4 \text{ kNm}$$

$$M_E = 2 \times 4 = 4 \text{ kNm}$$

$$M_G = 1 \times 4 - 1 \times 2 - 1 \times 1$$

$$= 4 - 2 - 1 = 1$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \alpha$$

$$I = \frac{bd^3}{12} = \frac{0.15 \times 0.25^3}{12}$$

$$= 0.0001953 \text{ m}^4$$

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M}{I} \times y$$

$$= \frac{750 \times 10^3}{0.0001953} \times 0.125$$

$$= 4.8 \times 10^8 \text{ N/m}^2$$

$$\frac{M}{I} = \frac{E}{R}$$

$$R = \frac{M}{I} \times E$$

$$= \frac{750 \times 10^3}{0.0001953} \times 200 \times 10^9$$

$$R = 52.08$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

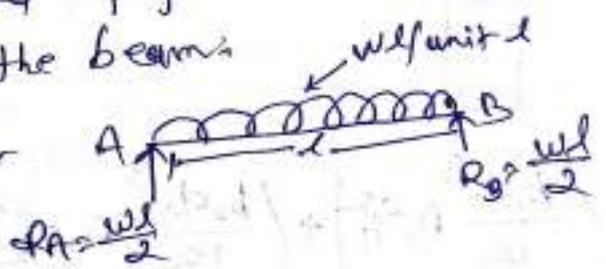
$$= \frac{750 \times 10^3}{0.0001953} \times 0.06$$

$$= 230.41 \times 10^6$$

$y = 0.0125 \text{ m}$
 $b = 0.15 \text{ m}$
 $d = 0.25 \text{ m}$

Q. A simply supported timber beam of rectangular section carries 80 kN/m uniform distributed load over a span of 3.6 m. If the depth of the beam is twice its width and maximum stress is not exceed 7 MPa. find the dimension of the beam.

Sol:



$$B.M = M_c = \text{load} \times \text{dis.} \times \frac{\text{dis.}}{2}$$

$$= w \times \frac{l}{2} \times \frac{l}{4}$$

$$M_{\text{max}} = \frac{wl^2}{8} \text{ or } \frac{wl}{8}$$

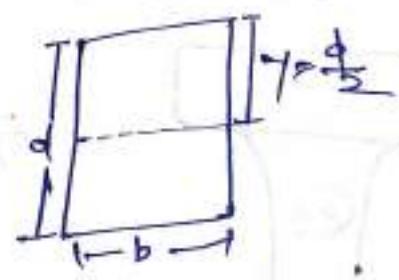
$$M = \frac{80 \times 3.6^2}{8}$$

$$M = 10.5 \text{ kN-m}$$

$$M = 10.5 \times 10^3 \text{ N-m}$$

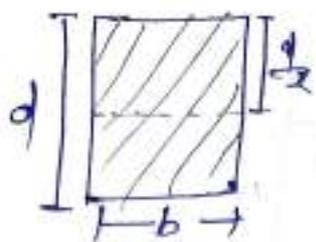
$$d = 2b$$

$$\sigma = 7 \text{ MPa} = 7 \times 10^6 \text{ N/m}^2$$



1) Moment of Inertia

Solid Rectangular Section, Solid Circular Section



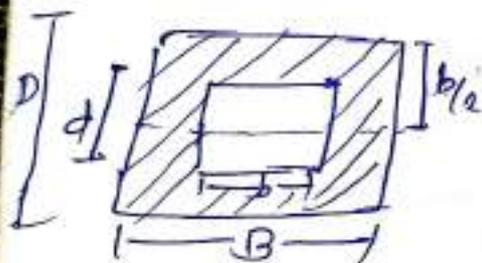
$$I = \frac{bd^3}{12}$$



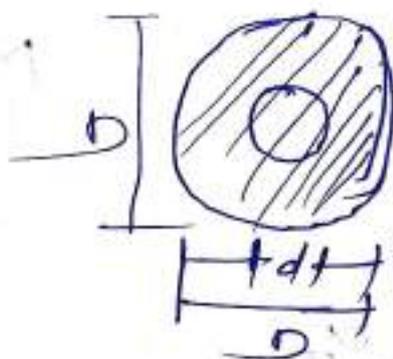
$$I = \frac{\pi d^4}{64}$$

Hollow Rectangular Section

Hollow Circular Section



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$



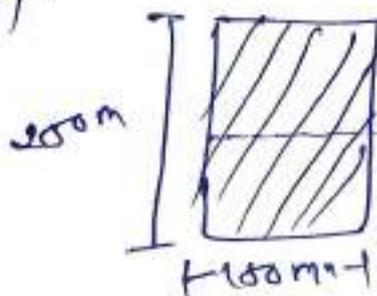
$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

Q. A 200 mm (depth) x 150 mm (width) rectangular beam is subjected to maximum bending moment of 750 kN-m. Determine:

1) maximum stress

2) If $E = 200 \text{ GPa}$; find radius of curvature

Sol.



$$r = \frac{200}{2} = 125$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$$

$$M = 750 \text{ kN-m} = 750 \times 10^3 \text{ N-m}$$

$$\sigma = \text{max}$$

$$R = ?$$

$$\gamma = \frac{125 - 65}{60}$$

$$\therefore \sigma = \frac{E}{R} \gamma$$

force in elementary strip
 $F = \frac{E}{R} \cdot \gamma \cdot \delta a$

Bending moment
 = force \times Distance
 $= \frac{E}{R} \cdot \gamma \cdot \delta a \cdot \gamma$

$$\Delta M_b = \frac{E}{R} \cdot \gamma^2 \cdot \delta a$$

$$\Sigma \Delta M_b = \frac{E}{R} \Sigma \gamma^2 \cdot \delta a$$

$$M = \frac{E}{R} \cdot I$$

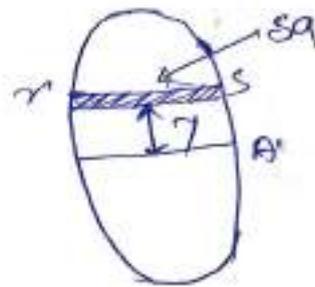
$$\boxed{\frac{M}{I} = \frac{E}{R}} \quad \text{--- (2)}$$

सभी 1 व 2 से

$$\frac{M}{I} = \frac{E}{R}, \quad \frac{\sigma}{\gamma} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{\gamma} = \frac{E}{R}}$$

Derivation of Bending moment
 in equation



$$M \cdot I = \text{Distance}^2 \times \text{Area}$$

$$I = \Sigma \gamma^2 \cdot \delta a$$

$$M = \Sigma \Delta M_b$$

M -

I -

σ - Stress

γ - Neutral axis of Distance

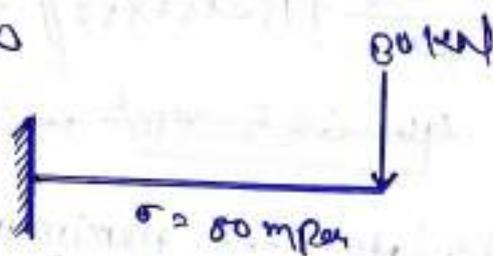
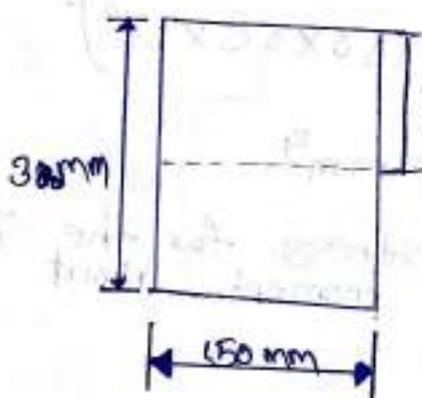
E - modulus of Elasticity

R - Radius of curvature,

$$\begin{aligned}\sigma &= \frac{M}{I} \times y \\ &= \frac{3.4 \times 10^3}{0.531 \times 10^{-4}} \times 0.125 \\ &= 8.0037 \times 10^6 \text{ N/m}^2\end{aligned}$$

Q. → A Cantilever beam of rectangular section 150 mm wide and 300 mm deep. If the cantilever is subjected to a point load of 30 kN at the free end and the bending stress is not to exceed 50 MPa. Find the span of the beam.

Sol:

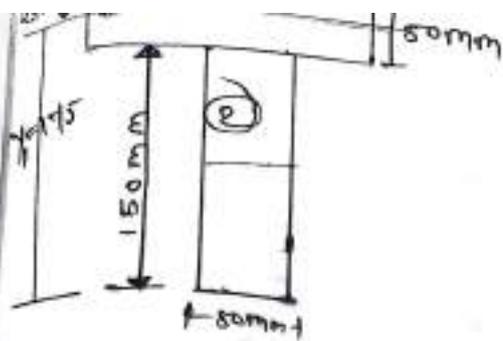


$$\begin{aligned}b &= 150 \text{ mm} = 150 \times 10^{-3} \text{ m} \\ d &= 300 \text{ mm} = 300 \times 10^{-3} \text{ m} \\ \sigma &= 50 \times 10^6 \text{ Pa}\end{aligned}$$

$$\text{सूत्र } \left[\frac{M}{I} = \frac{\sigma}{y} \right]$$

$$I = \frac{bd^3}{12} = \frac{(150 \times 10^{-3}) \times (300 \times 10^{-3})^3}{12}$$

$$I = 3.375 \times 10^{-4} \text{ m}^4$$



$$A_1 = 150 \times 15 \text{ mm}^2$$

$$y_1 = 15 = \frac{150}{20} = 7.5 = 175 \text{ mm}$$

$$A_2 = 150 \times 150 \text{ mm}^2$$

$$y_2 = \frac{150}{2} = 75 \text{ mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{150 \times 15 \times 175 + 150 \times 150 \times 75}{150 \times 15 + 150 \times 150} = 120 \text{ mm}$$

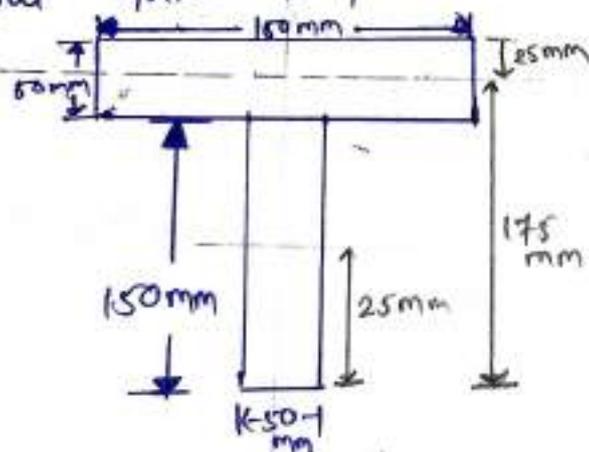
$$I = \left(\frac{b_1 d_1^3}{12} A_1 y_1^2 \right) + \left(\frac{b_2 d_2^3}{12} A_2 y_2^2 \right) \quad h_1 = (175 - 120) = 50$$

$$h_2 = (125 - 75) = 50$$

$$= \left(\frac{150 \times 15^3}{12} + 150 \times 150 \times 50^2 \right) + \left(\frac{150 \times 150^3}{12} + 150 \times 150 \times 50^2 \right)$$

$$= \cancel{40625000} \text{ mm}^4 = 5312500 \text{ mm}^4$$

Q. → Calculate the maximum bending stress for the T-section given below in the fig. take moment about Neutral axis = 3.4 kN-m.



$$P_{01} \therefore M = 3.4 \text{ kN-m} = 3.4 \times 10^3 \text{ kN/m}$$

$$I = 5312500 \text{ mm}^4 = 5312500 \times 10^{-12} \text{ m}^4 = 0.531 \times 10^{-6} \text{ m}^4$$

$$y = \bar{y} = 125 \text{ mm} = 0.125 \text{ m}$$

$$\left\{ \frac{M}{I} = \frac{\sigma}{y} \right\}$$

Q114 ∴ Solid rectangular section for

$$I = \frac{bd^3}{12} = \frac{b \times (2b)^3}{12} = \frac{8b^3}{12} = \frac{2b^3}{3}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{13.5 \times 10^3}{\frac{2b^3}{3}} = \frac{7 \times 10^6}{b}$$

$$\Rightarrow b^3 = \frac{13.5 \times 10^3 \times 3}{7 \times 10^6}$$

$$b^3 = 5.785 \times 10^{-3} \text{ m}$$

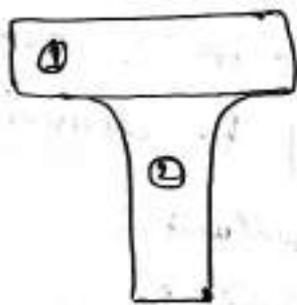
$$b = 0.00578 \text{ m}$$

$$d = 2b$$

$$d = 2 \times 0.00578$$

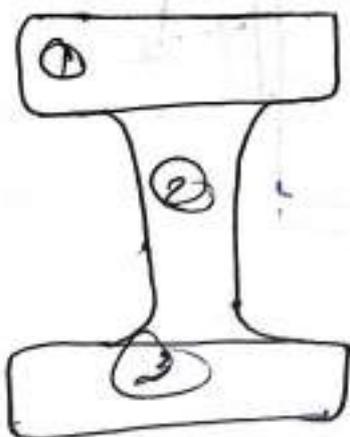
$$d = 0.01156 \text{ m}$$

T-Section and I-Section



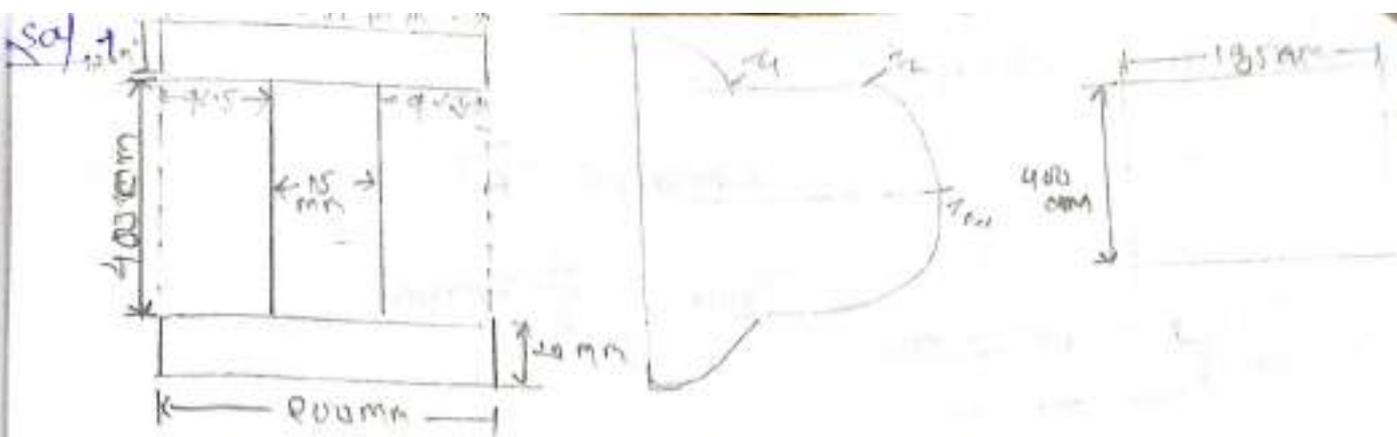
$$\bar{y} = \frac{A_1 \gamma_1 + A_2 \gamma_2}{A_1 + A_2}$$

$$I = \left(\frac{b_1 d_1^3}{12} A_1 \gamma_1^2 \right) + \left(\frac{b_2 d_2^3}{12} A_2 \gamma_2^2 \right)$$



$$\bar{y} = \frac{A_1 \gamma_1 + A_2 \gamma_2 + A_3 \gamma_3}{A_1 + A_2 + A_3}$$

$$I = \left(\frac{b_1 d_1^3}{12} A_1 \gamma_1^2 \right) + \left(\frac{b_2 d_2^3}{12} A_2 \gamma_2^2 \right) + \left(\frac{b_3 d_3^3}{12} A_3 \gamma_3^2 \right)$$



$$I = \frac{B D^3}{12} - \frac{b d^3}{12} = \frac{200 \times 440^3}{12} - \frac{185 \times 400^3}{12} = 433066666.67 \text{ m}^4$$

$$Z_1 = \frac{150}{8 \times 433066666.67} (0.44^2 - 0.4^2) = 1.454741379 \times 10^{-9} \text{ m/m}$$

$$Z_2 = Z_1 \frac{D}{b} = 1.454741379 \times 10^{-9} \times \frac{0.2}{0.185} = 1572693383 \times 10^{-9} \text{ m/m}$$

$$Z_3 = \frac{150}{8 \times 433066666.67 \times 10^{-9} \times 0.15} (0.2(0.44^2 - 0.4^2) + 0.015 \times 0.4^2)$$

$$= 2632.891 \text{ kN/m}^2$$

Q. → A laminated woodent beam 10cm wide and 15cm deep is made up of three 50cm wide planks glued together to resist the longitudinal shear. The beam is simply support over a span of 2m. If the glued joint is 0.45 MN/m². Find the safe concentrated load that the beam may carry at its centre

$$\text{Sol} \rightarrow I = \frac{b d^3}{12}$$

$$I = \frac{0.1 \times 0.15^3}{12}$$

$$I = 2.8125 \times 10^{-5} \text{ m}^4$$

$$= 0.45 \times 10^6 = \frac{w}{2} \frac{(0.1 \times 0.05)}{2.8125 \times 10^{-5} \times 0.1} (0.25^2 + 0.05^2)$$

$$= 10135 \text{ N}$$

$$I = \frac{S A_1 \bar{y}_1}{I_b} + \frac{S A_2 \bar{y}_2}{I_b}$$

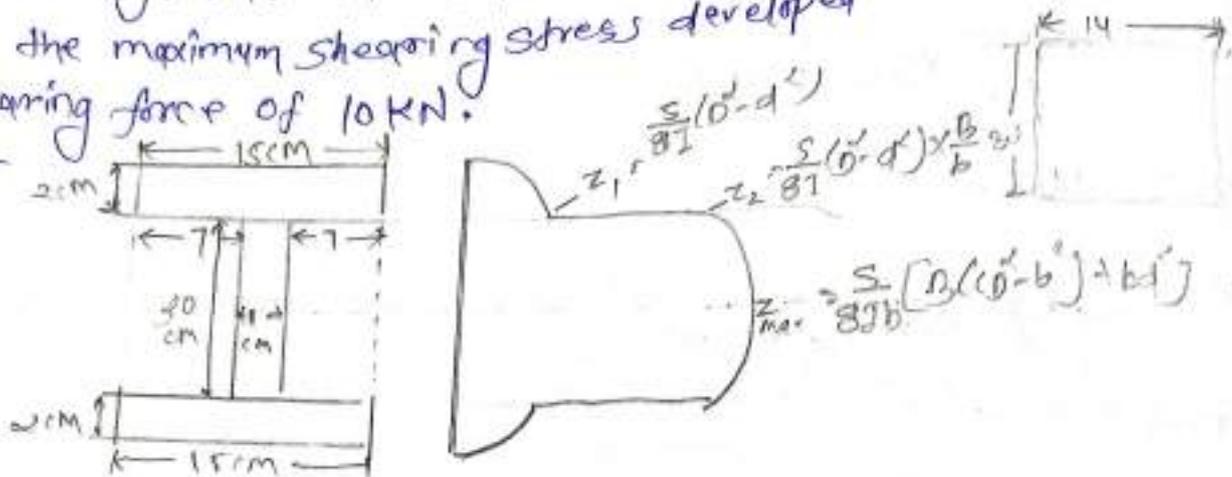
$$= \frac{S}{I_b} (A_1 \bar{y}_1 + A_2 \bar{y}_2)$$

Q. An I section, with rectangular ends, has the following dimensions:-

Flange: 15cm x 2cm, web: 30cm x 2cm

Find the maximum shearing stress developed in the beam for a shearing force of 10 kN.

Sol: -



$$I = \frac{BD^3 - bd^3}{12} \Rightarrow \frac{15 \times 34^3}{12} - \frac{14 \times 30^3}{12} \Rightarrow 17630 \text{ cm}^4$$

$$S = 10 \text{ kN}$$

$$D = 34 \text{ cm} = 0.34 \text{ m}$$

$$I = 17630 \times 10^{-8}$$

$$B = 15 \text{ cm} = 0.15 \text{ m}$$

$$\bar{y} = 17 \text{ cm}$$

$$b = 2 \text{ cm} = 0.02 \text{ m}$$

$$d = 30 \text{ cm} = 0.3 \text{ m}$$

$$z_1 = \frac{S}{8I} (D^2 - d^2) = \frac{10}{8 \times 17630 \times 10^{-8}} (0.34^2 - 0.3^2) = 181.509 \text{ kN/m}^2$$

$$z_2 = z_1 \times \frac{b}{B} = 181.509 \times \frac{0.02}{0.15} = 2417.16 \text{ kN/m}$$

$$z_{max} = \frac{S}{8Ib} [B(b^2 - d^2) + bd^2] = \frac{10}{8 \times 17630 \times 10^{-8} \times 0.02} [0.15(0.34^2 - 0.3^2) + 0.15 \times 0.3^2]$$

$$= 2360 \text{ kN/m}^2$$

Q. An I-section having flange 200mm x 20mm and web 400mm x 10mm is used as a beam. If the section is subjected to a shear force of 100 kN, find the shear stress in the beam and show the variation of shear stress across the section.

अब सूत्र $\frac{M}{I} = \frac{\sigma}{r}$ से

$$M = \frac{\sigma}{r} \times I$$

$$= \frac{50 \times 10^6}{150} \times 3.375 \times 10^4$$

$$M = 112.5$$

$$M = \text{load} \times \text{Distance}$$

$$112.5 = 30 \times l$$

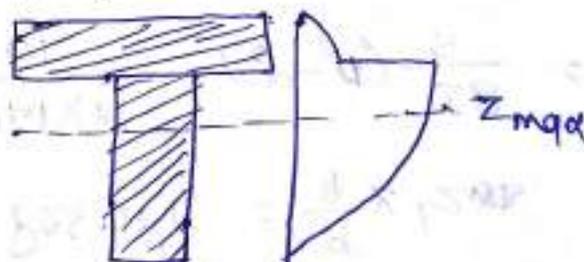
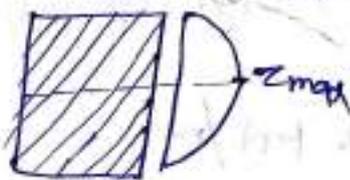
$$l = \frac{112.5}{30 \times 10^3}$$

$$l = 3.75 \text{ m}$$

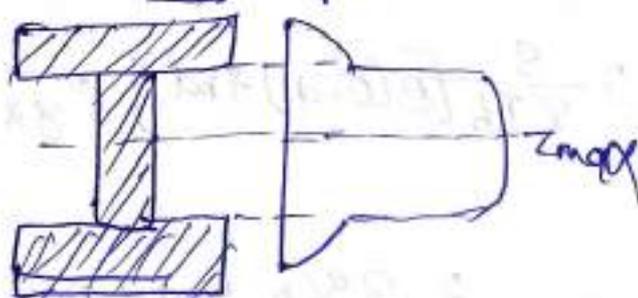
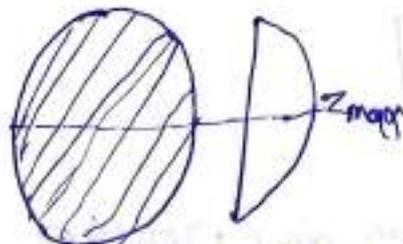
$$l = 3.75 \times 10^3 \text{ m}$$

SHEAR STRESS DISTRIBUTION

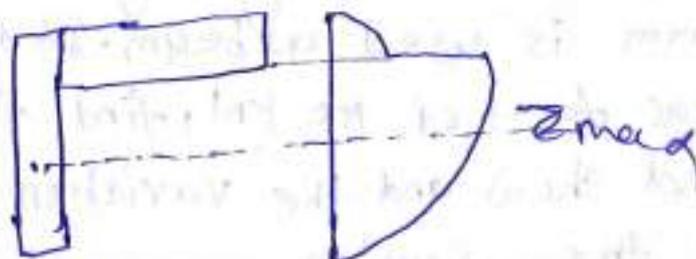
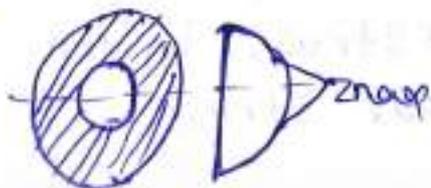
1) Solid Rectangular section (1) T-section.



2) Solid Circular section :- (2)



3) Hollow Circular section :- (3)



$$\tau_{max} = \frac{100 \times 0.6015625 \times 10^{-4}}{0.0001134 \times 0.05} = 1102.96 \text{ KN/m}^2$$

Shear stress in the flange at the junction of flange and web.

$$= \frac{100 \times 0.2 \times (0.0375 + 0.025)}{0.0001134 \times 0.2} = 2755 \text{ KN/m}^2$$

Shear stress in the web at the junction of flange and web.

$$\tau_2 = \frac{SA\bar{y}}{Ib}$$

$$\tau_2 = \frac{100 \times 0.2 \times 0.05 \times (0.0375 + 0.025)}{0.0001134 \times 0.05}$$

$$= 1102.96 \text{ KN/m}^2$$

Q. A beam of T-section has the flange 120 mm x 10 mm and web 100 mm x 10 mm subjected to a shear force 120 kN. Calculate the shear stress at the neutral axis and at the junction of the web and the flange.

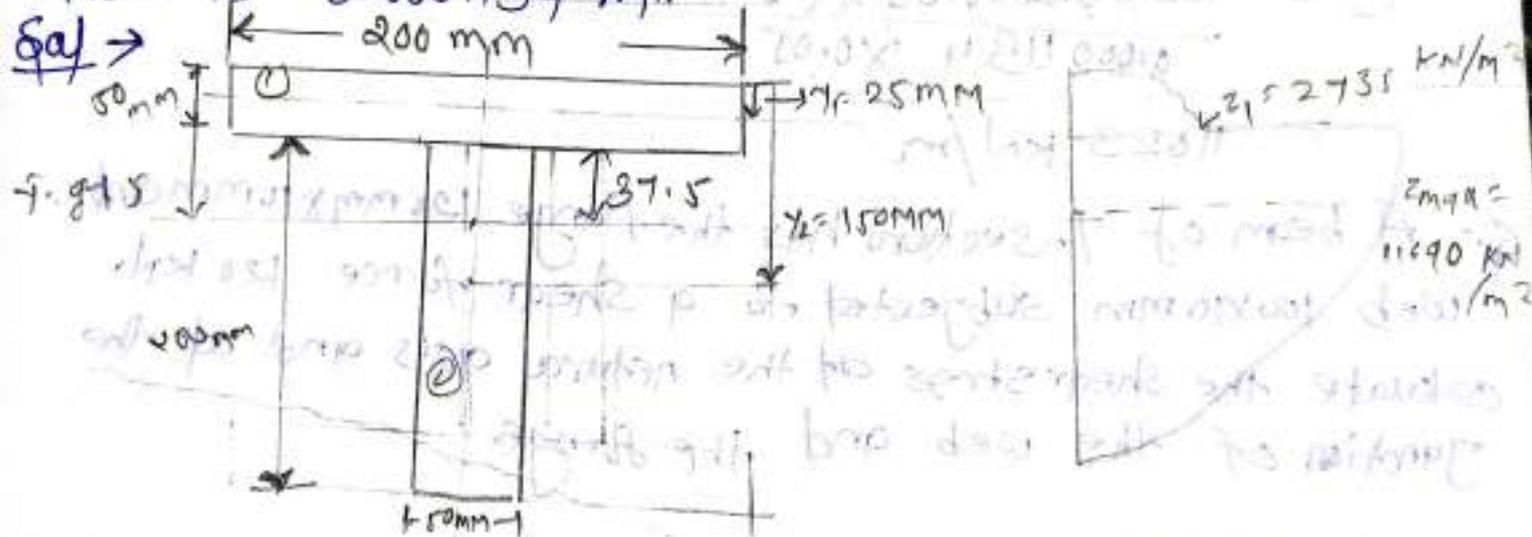
$$Z_{\text{mean}} = \frac{S}{A} = \frac{10}{7.92869 \times 10^3} = 1261.24 \text{ kN/m}^2$$

$$Z_{\text{max}} = \frac{4}{3} \times Z_{\text{mean}} = \frac{4}{3} \times 1261.24 = 1681.65 \text{ kN/m}^2$$



$$Z_{\text{max}} = 1681.65 \text{ kN/m}^2$$

Q. → A T-section is subjected to a vertical shear force of 100 kN. Calculate the shear stress at the neutral axis and at the junction of the web and flange. Moment of inertia about the horizontal neutral axis is 0.0001134 m⁴.



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(200 \times 25) \times 12.5 + (200 \times 150) \times 150}{(200 \times 25) + (200 \times 150)} = 37.5 \text{ mm}$$

Shear stress at neutral axis

$$Z_{\text{max}} = \frac{S A \bar{y}}{I_b}$$

$$A \bar{y} = (200 \times 25) \times 12.5 + (37.5 \times 200) \times 118.75$$

$$A \bar{y} = 61316.25 \text{ mm}^3$$

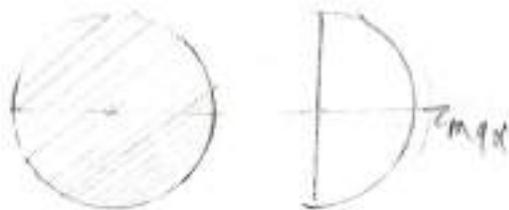
$$= 6.60156256 \times 10^{-4} \text{ m}^3$$

37.5
2

Circular section :-

$$Z_{\text{mean}} = \frac{S}{A}$$

$$Z_{\text{max}} = \frac{4}{3} Z_{\text{mean}}$$



Q. → A Circular beam 150mm diameter is subjected to a shear force of 7kN. Calculate the value of maximum shear stress and sketch variation of shear stress along the depth of the beam.

Sol: $d = 150 \text{ mm} = 0.15 \text{ m}$
 $S = 7 \text{ kN}$

$$A = \frac{\pi}{4} d^2$$
$$= 0.01767 \text{ m}^2$$

$$Z_{\text{mean}} = \frac{S}{A} = \frac{7}{0.01767} = 396.151 \text{ kN/m}^2$$

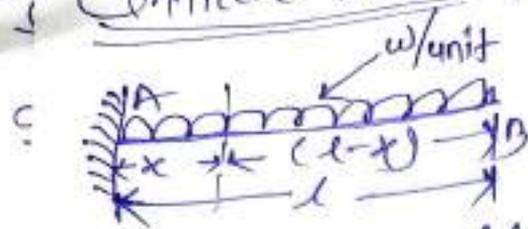
$$Z_{\text{max}} = \frac{4}{3} \times 396.151 = 528.201 \text{ kN/m}^2$$

Q. → The Circular beam of 100.5 mm diameter is subjected to a shear force of 10kN. Calculate the value of mean and maximum shear stress and sketch the variation of the shear stress along the depth of the beam.

Sol: $d = 100.5 \text{ mm} = 0.1005 \text{ m}$
 $S = 10 \text{ kN}$

$$A = \frac{\pi}{4} d^2$$
$$= \frac{\pi}{4} (0.1005)^2 = 7.92869 \times 10^{-3} \text{ m}^2$$

Concave beam uniform load



$$M = -w \cdot (l-x) \cdot \frac{(l-x)}{2}$$

$$M = -\frac{w(l-x)^2}{2}$$

$$EI \frac{d^2y}{dx^2} = -\frac{w(l-x)^2}{2}$$

on Integrating

$$\int EI \frac{dy}{dx} = \int -\frac{w(l-x)^2}{2}$$

$$EI \frac{dy}{dx} = -\frac{w(l-x)^3}{6} + C_1 \quad \text{--- (1)}$$

At Point A

$$x=0, \frac{dy}{dx} = 0$$

$$C_1 = \frac{wl^3}{6}$$

$$EI \frac{dy}{dx} = -\frac{w(l-x)^3}{6} + \left(\frac{wl^3}{6}\right) \quad \text{--- (2)}$$

At Point B

$$x=l$$

$$EI \cdot \frac{dy}{dx} = 0 - \frac{wl^3}{6}$$

$$\frac{dy}{dx} = -\frac{wl^3}{6EI}$$

$$\theta_B = -\frac{wl^3}{6EI}$$

Slope at point B $x=l$

$$EI \frac{dy}{dx} = -w(l-l - \frac{l}{2})$$

$$EI \frac{dy}{dx} = -w(l - \frac{l}{2})$$

$$EI \frac{dy}{dx} = \frac{-wl}{2} \quad \underline{\text{Double Integration Method:}}$$

$$\theta_B = \frac{dy}{dx} = \frac{-wl}{2}$$

$$\boxed{\theta_B = \frac{-wl}{2}}$$

$$EI \frac{dy}{dx} = -w(lx - \frac{x^2}{2})$$

on integration

$$\int EI \frac{dy}{dx} = \int -w(lx - \frac{x^2}{2})$$

$$EI \cdot y = -w \left(\frac{lx^2}{2} - \frac{x^3}{6} \right) + C_2 \quad \text{--- (3)}$$

At point A

$$x=0, y=0$$

$$0 = C_2$$

$$EI \cdot y = -w \left(\frac{lx^2}{2} - \frac{x^3}{6} \right)$$

Deflection of point B

$$EI \cdot y = -w \left(\frac{l^3}{2} - \frac{l^3}{6} \right)$$

$$\boxed{y = -\frac{wl^3}{3EI}}$$

Salut

Chapter - 4 Deflection of beam

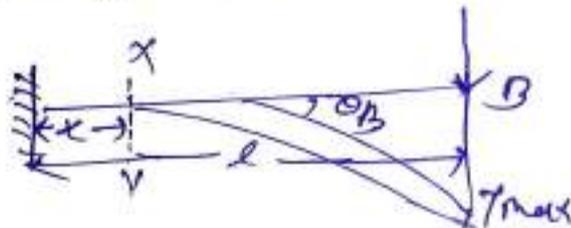
slope and deflection



Relation between Radius of curvature, slope and deflection -

formula $\rightarrow M = EI \frac{d^2y}{dx^2}$ \rightarrow Derivation is No syllabus.
slope = $\frac{dy}{dx}$, deflection = δ

Cantilever beam :-



$$M = -w(l-x)$$

$$EI \frac{d^2y}{dx^2} = -w(l-x)$$

on Integrating

$$\int EI \frac{dy}{dx} = \int -w(l-x)$$

$$EI \frac{dy}{dx} = -w(lx - \frac{x^2}{2}) + C_1 \quad \text{--- (1)}$$

At point A

$$x=0, \frac{dy}{dx} = 0$$

on putting value of $x=0$ and $\frac{dy}{dx} = 0$

$$0 = C_1$$

$$EI \frac{dy}{dx} = -w(lx - \frac{x^2}{2}) \quad \text{--- (2)}$$

Slope at point C

$$x = \frac{l}{2}, \gamma_c = \frac{d\gamma}{dx}$$

$$EI \gamma_c = \frac{w(\frac{l}{2})^2}{4} - \frac{wl^2}{16}$$

$$\gamma_c = 0$$

Slope at A

$$x = 0$$

$$EI \gamma_A = -\frac{wl^2}{16}$$

$$\boxed{\gamma_A = -\frac{wl^2}{16EI}}$$

$$\int EI \frac{d\gamma}{dx} = \int \frac{wx^2}{4} - \frac{wl^2}{16}$$

$$EI \gamma = \frac{wx^3}{12} - \frac{wl^2}{16}x + C_2$$

$$x=0, \gamma=0, C_2=0$$

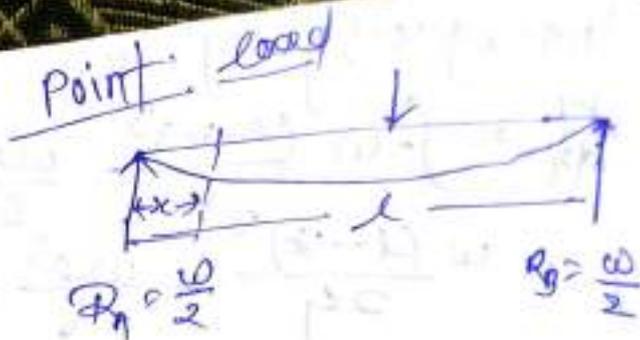
$$EI \gamma = \frac{wx^3}{12} - \frac{wl^2}{16}x$$

$$\gamma_c = \frac{w(\frac{l}{2})^3}{12} - \frac{wl^2}{16} \cdot (\frac{l}{2})$$

$$= \frac{wl^3}{96} - \frac{wl^3}{32} = -\frac{wl^3}{48}$$

$$\boxed{\gamma_c = -\frac{wl^3}{48EI}}$$

Simply supported beam:-



$$M = \frac{w}{2}(l-x) - w\left(\frac{l}{2} - x\right)$$

$$= \frac{wl}{2} - \frac{wx}{2} - \frac{wl}{2} + wx$$

$$= -\frac{wx}{2} + wx$$

$$EI \frac{d^2y}{dx^2} = \frac{wx}{2} + wx = \frac{3wx}{2}$$

on Integrating

$$\int EI \frac{d^2y}{dx^2} = \int \frac{3wx}{2} + wx$$

$$EI \frac{dy}{dx} = \frac{w}{2} \cdot \frac{x^2}{2}$$

$$EI \frac{dy}{dx} = \frac{wx^2}{4} + C \quad \text{--- (1)}$$

At point C

$$x = \frac{l}{2}, \frac{dy}{dx} = 0$$

$$0 = \frac{w\left(\frac{l}{2}\right)^2}{4} + C$$

$$C = -\frac{wl^2}{16}$$

$$EI \frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl^2}{16} \quad \text{--- (2)}$$

on Integrating eq. 2.

$$\int EI \frac{d^2 y}{dx^2} = \int -w \left(\frac{(l-x)^3}{6} - \frac{wl^3}{6} \right)$$

$$EI \cdot y = w \frac{(l-x)^2}{24} - \frac{wl^3}{6} \cdot x + C_2 \quad (3)$$

At point A

$$x=0, y=0$$

$$0 = \frac{-wl^4}{24} + C_2$$

$$C_2 = \frac{wl^4}{24}$$

$$EI \cdot y = \frac{-wl(l-x)^4}{24} - \frac{wl^3}{6}x + \frac{wl^4}{24}$$

Deflection at point B
 $x=l$

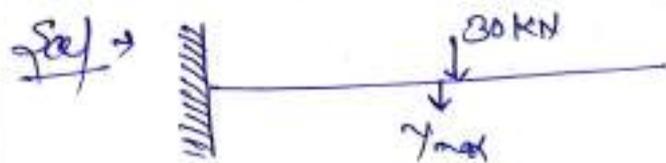
$$EI \cdot y = 0 - \frac{wl^4}{6} + \frac{wl^4}{24}$$

$$EI \cdot y = -\frac{wl^4}{8}$$

$$y_B = -\frac{wl^4}{8EI}$$

Deflection on point B.

Q → A steel cantilever of span 2.5m carrying a point load of 30kN at its free end. The moment of inertia of the cantilever beam is 9900 cm⁴. Find the maximum deflection of beam at the free end. Deflection
 • Take $E = 210 \text{ GN/m}^2$.



$$I = 9900 \text{ cm}^4 = 9900 \times 10^{-8} \text{ m}^4$$

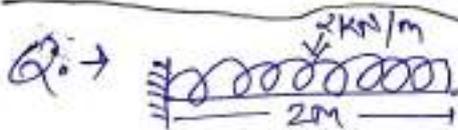
$$E = 210 \text{ GN/m}^2 = 210 \times 10^9 \text{ N/m}^2$$

$$W = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$l = 2.5 \text{ m}$$

$$\text{Slope } \theta_B = -\frac{wl^2}{2EI} = \frac{30 \times 10^3 \times (2.5)^2}{2 \times 210 \times 10^9 \times 9900 \times 10^{-8}} = 4.509 \times 10^{-3} \text{ radian}$$

$$\gamma_{max} = -\frac{wl^3}{3EI} = \frac{30 \times 10^3 \times (2.5)^3}{3 \times 210 \times 10^9 \times 9900 \times 10^{-8}} = 7.515 \times 10^{-3} \text{ m}$$



100 mm wide
 240 mm deep
 $E = 210 \text{ GN/m}^2$
 $\theta_B = ?$
 $\gamma_{max} = ?$

$$I = \frac{bd^3}{12}$$

$$= \frac{0.1 \times 0.24^3}{12}$$

$$= 1.152 \times 10^{-4}$$

on Integrating eqn - (2)

$$EI \cdot \gamma = \frac{w \cdot x^3}{12} - \frac{w x^4}{24} - \frac{w l^3}{24} x + C_2 \quad - (3)$$

At point A

$$x = 0, \gamma = 0$$

$$0 = 0 - 0 - 0 + C_2$$

$$C_2 = 0$$

$$EI \cdot \gamma = \frac{w l x^3}{12} - \frac{w x^4}{24} - \frac{w l^3}{24} \cdot x \quad - (4)$$

At point A

$$\gamma_A = 0$$

At point C

$$x = \frac{l}{2}$$

$$\text{eqn (4) } x = \frac{l}{2}$$

$$EI \gamma = \frac{w l^4}{96} - \frac{w l^4}{384} - \frac{w l^4}{48}$$

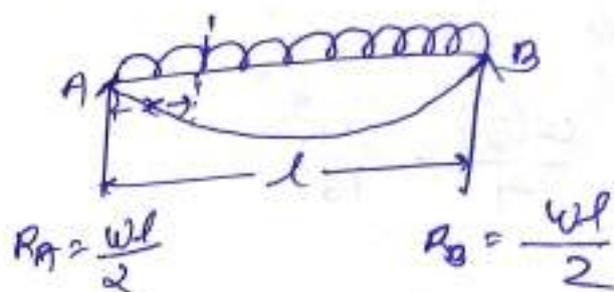
$$EI \gamma = \frac{-5 w l^4}{384}$$

$$\gamma_c = \frac{-5 w l^4}{384 EI}$$

Simply supported beam (uniform load)

$$M = \frac{wl}{2}x - wx \cdot \frac{x}{2}$$

$$M = \frac{wl}{2}x - \frac{wx^2}{2}$$



$$EI \frac{dy}{dx} = \frac{wl}{2}x - \frac{wx^2}{2}$$

On Integrating

$$EI \frac{dy}{dx} = \frac{wl}{4}x^2 - \frac{wx^3}{6} + C_1 \quad \text{--- (1)}$$

At point C

$$x = \frac{l}{2}, \quad \frac{dy}{dx} = 0$$

$$0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$C_1 = \frac{wl^3}{48} - \frac{wl^3}{16}$$

$$C_1 = -\frac{wl^3}{24}$$

$$EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24} \quad \text{--- (2)}$$

Slope At point A

$$x = 0$$

$$EI \frac{dy}{dx} = -\frac{wl^3}{24}$$

$$\frac{dy}{dx} = \frac{wl^3}{24EI}$$

$$\theta_A = -\frac{wl^3}{24EI}$$

safe. compressive load;
 $s.c.l = \frac{\text{criping load}}{f.o.s \text{ (force of safety)}}$

Q.7 A solid round bar 60mm in diameter in diameter and 2.5m long is used as a strut, one end of the strut is fixed, while the other end is hinged. Find the safe compressive load for this strut, using Euler's formula. Assume $E = 200 \text{ GN/m}^2$ and factor of safety 3.

Sol: $P_c = \frac{\pi^2 EI}{l_e^2}$

$E = 200 \text{ GN/m}^2 = 200 \times 10^9 \text{ N/m}^2$
 $D = 60 \text{ mm} = 0.06 \text{ m}$
 $l = 2.5 \text{ m}$

one end fixed and other end is hinged -

$l_e = \frac{l}{\sqrt{2}} = \frac{2.5}{\sqrt{2}} = 1.768 \text{ m}$

$I = \frac{\pi}{64} \times d^4$

$= \frac{\pi}{64} \times 0.06^4 = 630 \times 10^{-9}$

$P_c = \frac{\pi^2 \times 200 \times 10^9 \times 630 \times 10^{-9}}{1.768^2} = 401.7 \times 10^3 \text{ N}$

$s.c.l = \frac{\text{criping load}}{f.o.s} = \frac{401.7 \times 10^3}{3} = 133.9 \times 10^3 \text{ N}$

Defination of the following: -

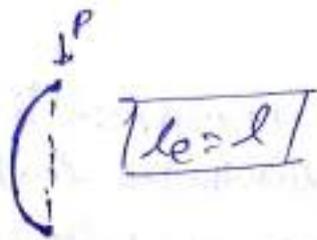
1) column

2) strut

3) slenderness ratio (k) = $\frac{\text{unsupported length}}{\text{min. radius of gyration}}$

Chapter 07 Column & strut

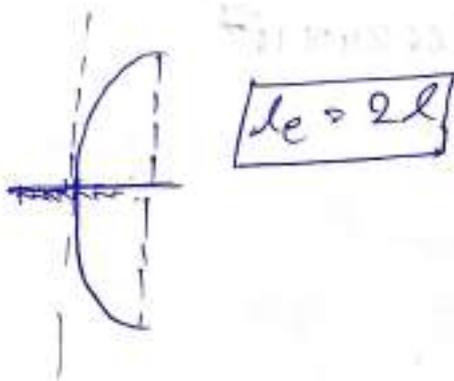
① Both end Hinged or pin



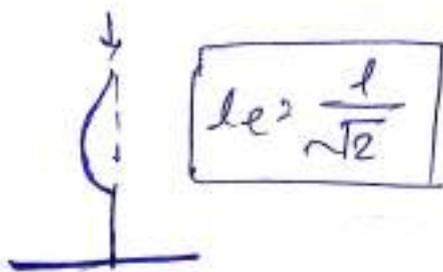
Euler's formula -

$$P_c = \frac{\pi^2 EI}{l_e^2}$$

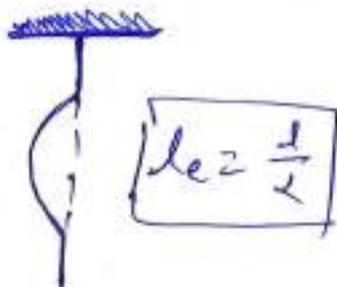
2) one end fix in other end free -



3) one end fix in other end pin joint -



4) Both ends are fixed



$$\theta_B = \frac{-wL^3}{6EI} = \frac{-2 \times 10^3 \times 2^3}{6 \times 210 \times 10^9 \times 115 \times 10^{-4}} = -1.102 \times 10^{-9} \text{ radian}$$

$$\tau_{max} = \frac{-wL^4}{8EI} = \frac{-2 \times 10^3 \times 2^4}{8 \times 210 \times 10^9 \times 115 \times 10^{-4}} = -1.653 \times 10^{-4} \text{ m}$$

Q. → A 250 mm long cantilever of rectangular section 48 mm and 36 mm deep carries a uniformly distributed load. Calculate the value of load w . If maximum deflection is 2 mm.

Take $E = 70 \text{ GN/m}^2$.

Solⁿ

$$I = \frac{bd^3}{12} = \frac{0.048 \times 0.036^3}{12} = 1.86624 \times 10^{-7} \text{ m}^4$$

$$0.002 = \frac{-w \times 250 \times 10^{-3}}{8 \times 70 \times 10^9 \times 1.86624 \times 10^{-7}}$$

$$w = 2.3921 \times 10^{-3} \text{ N/mm}$$

$$5000 \text{ cm}^2$$



CHAPTER - 8

$$\frac{T}{I_p} = \frac{z}{R} = \frac{C}{l}$$

$$T = \text{Torque} = z \cdot \frac{\tau}{16} D^2 (N-m)$$

$$I_p = \text{Polar moment of Inertia} = \frac{\pi}{32} d^4$$

$z =$ shear stress

$C =$ shear modulus

$\theta =$ angle of twist

$R =$ Radius

$$\text{Power } P = \frac{2\pi NT}{60} \quad \begin{array}{l} T = \text{mean or average torque} \\ N = \text{speed R.P.M.} \end{array}$$

Hollow Circular Shaft.

$$T = z \cdot \frac{\tau}{16} \left(\frac{D^4 - d^4}{D} \right) \quad \text{max } = \tau$$

$$I_p = \frac{\pi}{32} (D^4 - d^4)$$

Q. - A hollow shaft is to transmit 30 kW at 80 r.p.m. If the shear stress is not to exceed 60 MN/m² and internal diameter is 0.6 the external diameter. find the internal & external diameter assuming that the maximum torque is 1.4 times the mean.

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$= \frac{\pi}{64} (0.2^4 - 0.16^4)$$

$$= 4.637 \times 10^{-5}$$

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$= \frac{\pi}{4} (0.2^2 - 0.16^2)$$

$$= 0.11030973355$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.637 \times 10^{-5}}{0.11030973355}} = 0.02050272785$$

*) Buckling load

*) Classification of load

I) short column + जब length $l < 8d$ जे कम स या
 $k < 32$

II) medium size column + इसका length 8 से 30 गुना होता है व $k 32$ से 120 के बीच होगा।

III) long column - $l > 30d$, $k > 120$

Pankinels formula :-

$$P_R = \frac{\sigma_c \times A}{1 + a \left(\frac{l_e}{k} \right)^2} \quad \left\{ \because a = \frac{\sigma_c}{\pi^2 E} \right\}$$

σ_c = Compressive stress

l_e = equivalent length / effective length

k = radius of gyration

$$k = \sqrt{\frac{I}{A}}$$

Q.- A hollow C.I column whose outside dia. is 200mm has a thickness 20mm. It is 4.5m long and is fixed at both ends. calculate the safe load by using Pankinels formula

Take $\sigma_c = 550 \text{ MN/m}^2$, $a = \frac{1}{1600}$

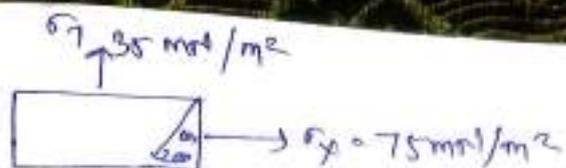
Sol. $\sigma_c = 550 \text{ MN/m}^2 = 550 \times 10^6 \text{ N/m}^2$

$d = 200 \text{ mm} = 0.2 \text{ m}$

$t = 20 \text{ mm} = 0.02 \text{ m}$

inner diameter = $d - 2t = 200 - 40 = 160 = 0.16 \text{ m}$

Solⁿ



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

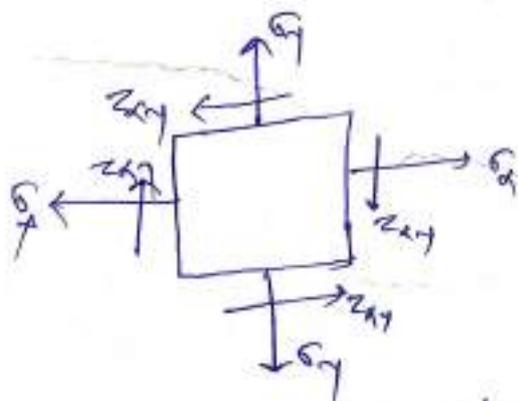
$$= \frac{75 + 35}{2} + \frac{75 - 35}{2} \cos 40^\circ = 70.92 \text{ MPa/m}^2$$

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$= \frac{75 - 35}{2} \sin 40^\circ = 12.85 \text{ MPa/m}^2$$

$$\sigma_p = \sqrt{\sigma_n^2 + \tau^2} = \sqrt{(70.92)^2 + (12.85)^2} = 71.48 \text{ MPa/m}^2$$

$$\tan \phi = \frac{\tau}{\sigma_n} = \frac{12.85}{70.92} = 0.1828, \quad \phi = 10.21^\circ \text{ (Ans)}$$



① major principal stress

$$\sigma_1 = \sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

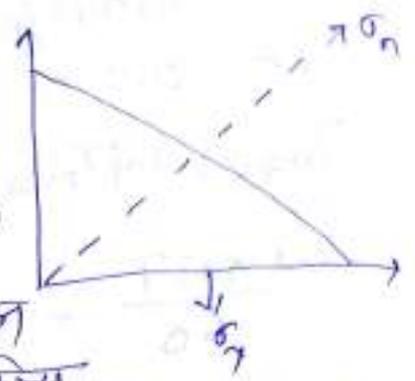
② minor principal stress

$$\sigma_2 = \sigma_{\min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

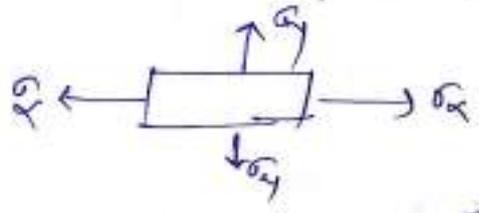
Chapter 5 Principle Stress

Q. - what is principle stress?

Ans. - किसी बिंदु पर कार्यरत अधिकतम और न्यूनतम सामान्य प्रतिबल होते हैं। जब किसी कण पर बल लागू होता है, तो उसके अंदर विभिन्न दिशाओं में प्रतिबल उत्पन्न होते हैं। प्रिंसिपल स्ट्रेस के दिशाएँ होती हैं जहाँ केवल सामान्य प्रतिबल होते हैं और कोई भी अपरूपण प्रतिबल नहीं होता है।



Two mutually perpendicular direct stress -



1) normal stress = $\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos^2 \theta$

2) shear stress or tangential stress (τ) = $\frac{\sigma_x - \sigma_y}{2} \sin 2\theta$

1) $\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$

4) Resultant stress (σ_r) = $\sqrt{\sigma_n^2 + \tau^2}$

5) $\tan \theta = \frac{\tau}{\sigma_n}$

Q. - The principle stress at a point across two perpendicular plane are 75 MN/m² (tensile) and 35 MN/m² (tensile). find the normal stress tangential stress and resultant stress and its obliquity on a plane at 20° with the major principal plane.

solⁿ → $P = 300 \text{ kW} = 300 \times 10^3 \text{ W}$
 $N = 80 \text{ r.p.m}$
 $Z = 60 \text{ MN/m}^2 = 60 \times 10^6 \text{ N/m}^2$
 $d = 0.6 D$

$$T_{\max} = 1.4 T_{\text{mean}}$$

$$P = \frac{2\pi NT}{60} = 300 \times 10^3 = \frac{2\pi \times 80 \times T}{60}$$

$$T = T_{\text{Avg.}} = 35809 \text{ N}\cdot\text{m}$$

$$T_{\max} = 1.4 \times 35809 = 50132 \text{ N}\cdot\text{m}$$

$$T_{\max} = Z \cdot \frac{\tau}{16} \left(\frac{D^4 - d^4}{D} \right)$$

$$50132 = 60 \times 10^6 \times \frac{\tau}{16} \left(\frac{D^4 - (0.6D)^4}{D} \right)$$

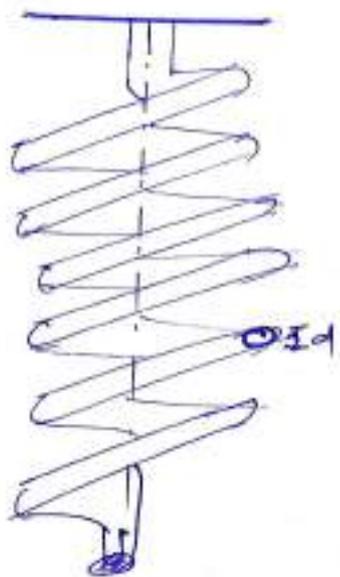
$$50132 = 60 \times 10^6 \times \frac{\tau}{16} D^3 (1 - 0.6^4)$$

$$D = 0.169 \text{ m}$$

$$d = 0.6 \times 0.169 = 0.102 \text{ m}$$

SPRING Chr 6

The close-coiled HELICAL SPRINGS:-



r = Radius of coil
 d = diameter of the wire of the coil

s = Deflection of coil under the load w

C = modulus of rigidity

n = numbers of coil or turns

θ = Angle of twist

l = Length of wire = $2\pi Rn$

τ = Shear stress, and

I_p = Polar moment of inertia

$$= \frac{\pi}{32} d^4$$

$$\frac{T}{I_p} = \frac{C\theta}{l} = \frac{\tau}{R}$$

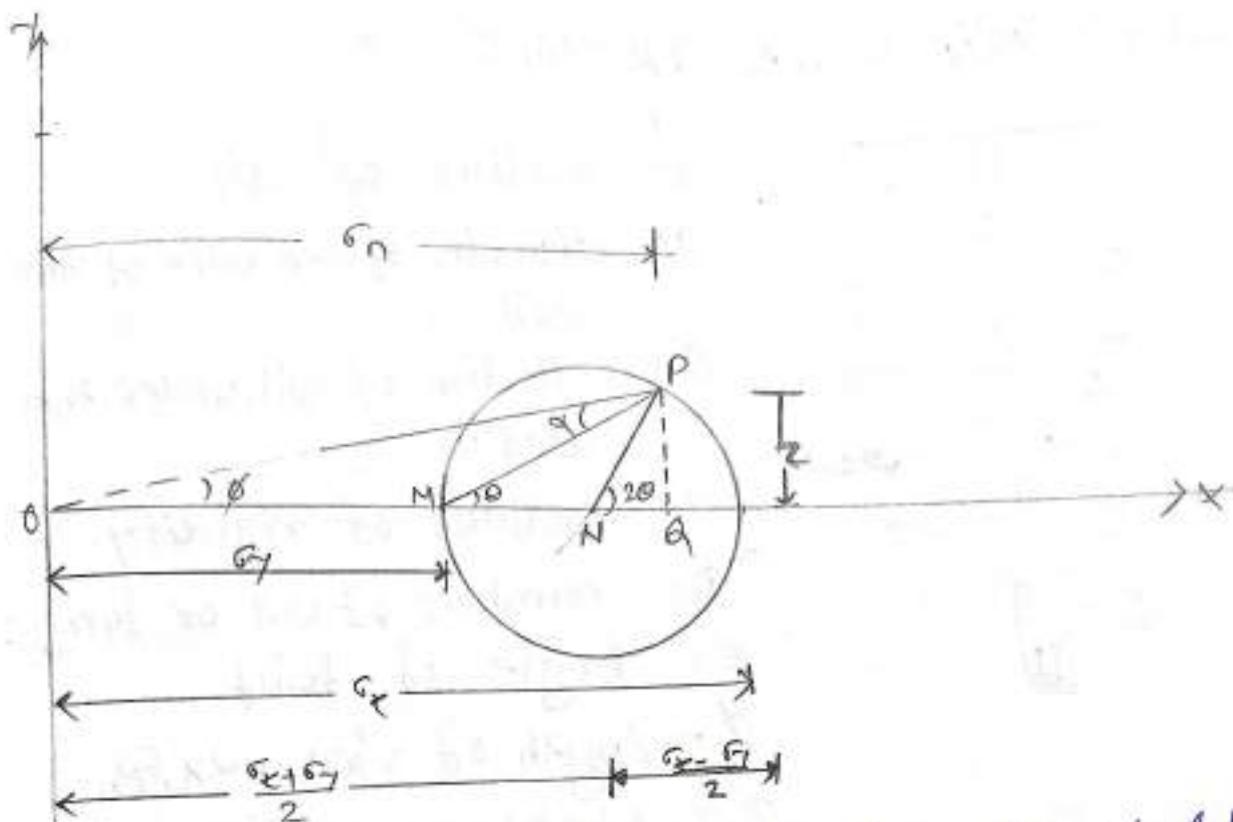
1) Shear stress $\tau = \frac{16WR}{\pi d^3}$

2) Deflection (s) = $\frac{64WR^3n}{Cd^4}$

3) Stiffness (k) = $\frac{W}{s}$

4) Energy store by the spring $U = \frac{1}{2} Ws$ (N-m) J

Mohr's Circle Construction for "Like Stresses".



load, Area, Inclination Angle At oblique

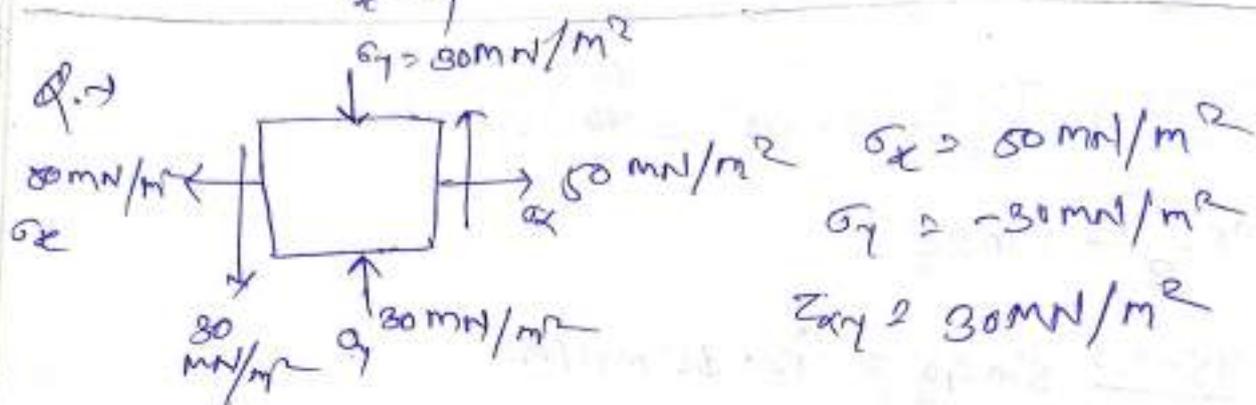
1) Normal stress = $\sigma_n = \frac{P}{A} \cos^2 \theta$

2) maximum shear stress $z_{max} = \frac{P}{2A}$

3) $z = \frac{P}{A} \cos \theta \sin \theta$

$$(3) \sigma_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$(4) \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



(1) major principle stress

$$\sigma_1 = \frac{50 + (-30)}{2} + \sqrt{\left(\frac{50 + 30}{2}\right)^2 + 30^2} = 60$$

(2) minor principle stress

$$\sigma_2 = \frac{50 + (-30)}{2} - \sqrt{\left(\frac{50 + 30}{2}\right)^2 + 30^2} = -40$$

$$3) \sigma_{max} = \frac{60 + 40}{2} = 50$$

$$4) \tan 2\theta = \frac{2 \times 30}{50 + 30} = \frac{60}{80} = \frac{3}{4}$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = \frac{36.87}{2}$$

$$2\theta = \tan^{-1} \frac{3}{4}$$

$$\theta = 18.435$$

$$\theta = 36.87$$

$$W = 200 \text{ N}; C = 80 \text{ GN/m}^2$$

deflection of the spring

$$s = \frac{64WR^3n}{Cd^4}, s = \frac{64 \times 200 \times (0.06)^3 \times 10}{80 \times 10^9 \times (0.01)^4}$$

$$= 0.03456 \text{ m} = 34.56 \text{ mm}$$

stiffness of the spring k:

$$k = \frac{W}{s}, k = \frac{200}{0.03456} = 579 \text{ N/mm}$$

maximum shear stress τ :

$$\tau = \frac{16WR}{\pi d^3} = \tau = \frac{16 \times 200 \times 0.06}{\pi \times (0.01)^3}$$

$$= 61.11 \times 10^6 \text{ N/m}^2$$

$$= 61.11 \text{ MN/m}^2$$

strain energy stored U:

$$U = \frac{1}{2} \cdot W \cdot s, = \frac{1}{2} \times 200 \times 0.03456$$

$$= 3.456 \text{ N/m}$$

Q. → A closed coiled spring is made of 10 mm diameter wire. It has 10 coils. The mean coil diameter is 80 mm. If it is loaded by 200 N axial load, find the following:

- ① stress induced in the spring wire
- ② stiffness of spring
- ③ axial deflection of the spring
- ④ the strain energy in spring

$$\text{Take } G = 0.84 \times 10^5 \text{ N/m}^2$$

Sol. $d = 10 \text{ mm}$

$n = 10$

$D = 80 \text{ mm}$

$W = 200 \text{ N}$

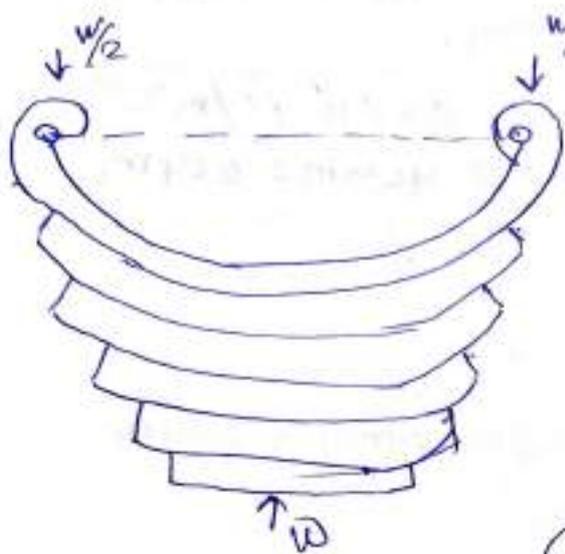
Stress induced in the spring wire $\tau = \frac{8WD}{\pi d^3}$

$$= \frac{8 \times 200 \times 80}{\pi \times 10^3}$$

$$= 40.76 \text{ N/mm}^2$$

Laminated spring :-

leaf spring, semi-elliptical spring, carriage spring.



$$\sigma_b = \frac{3wl}{2Nbt^2}$$

N = Number of plate

b = width

t = thickness

② Radius of curvature

$$\frac{\sigma_b}{\gamma} = \frac{E}{R}$$

③ Central deflection

$$\delta = \frac{3wl^3}{8ENbt^3}$$

Moment of each plates

$$m = \frac{wl}{4N}$$

Nov-Dec 2022

Ex: - A close-coiled helical spring is made out of 10 mm diameter steel rod. The coil consists of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 2000 N. Find the maximum shear stress induced in the section of the rod. If $C = 80 \text{ GPa/m}^2$. Find the deflection in the spring, the stiffness and strain energy in the spring.

Solution :- $d = 10 \text{ mm} = 0.01 \text{ m}$
 $n = 10$

$$R = \frac{120}{2} = 60 \text{ mm} = 0.06 \text{ m}$$

Q. A closed coil helical spring is made of 12mm diameter steel wire wound on a 10mm diameter mandrel. If there are 10 active coils what is spring constant? Take $C = 82 \text{ GPa/m}^2$. What force must be applied to the spring to elongate it by 40mm.

Solⁿ

$$d = 12 \text{ mm} = 0.012 \text{ m}$$

$$D = 10 \text{ mm}$$

$$R = 6 \text{ mm} = 0.006 \text{ m}$$

$$n = 10$$

$$k = ?$$

$$C = 82 \times 10^9 \text{ N/m}^2$$

$$S = 40 \text{ mm} = 0.04 \text{ m}$$

$$W = ?$$

Elongation of the Spring $S = 40 \text{ mm} = 0.04 \text{ m}$

Spring Constant:-

We know that

Spring constant = stiffness of spring (K)

$$K = \frac{W}{S} = \frac{Cd^4}{64R^3n} = \frac{82 \times 10^9 \times (0.012)^4}{64 \times \left[\frac{0.012}{2}\right]^3 \times 10}$$

$$K = 12300 \text{ N/m}$$

Force to be applied to the spring, $W =$

$$\frac{W}{S} = 12300 \text{ or } \frac{W}{0.04} = 12300$$

$$W = 492 \text{ N} \quad \underline{\text{Ans}}$$

$$P = \frac{3WL}{2nbt^2}$$

$$n = \frac{3WL}{20bt^2}$$

$$= \frac{3 \times 4 \times 10^3 \times 1}{2 \times 10 \times 0.09 \times 0.015^2} = 9.877$$

$$= \frac{3 \times 10^6}{29629629.168}$$

$$\delta = \frac{3WL^2}{8nEt^3} = \frac{3 \times 4 \times 10^3 \times (1)^3}{8 \times 10 \times 2 \times 10^{11} \times 0.09 \times (0.015)^3}$$

$$= 2.469 \times 10^{-3} \text{ m}$$

April-May
2024

Q. → A laminated spring 1 m long is made up of plates each 90 mm wide and 15 mm thick. If the bending stress in a plate is limited to 30 MN/m². How many plates are required to enable the spring to carry a central point load 3.5 kN? If E = 200 GN/m², what is the deflection under the load?

Sol → L = 1 m

b = 90 mm = 0.09 m

t = 15 mm = 0.015 m

W = 3.5 kN = 3500 N
= 3.5 × 10³ N

P = 30 MN/m² = 30 × 10⁶ N/m²

E = 200 GN/m² = 200 × 10⁹ N/m²

$$P = \frac{3WL}{2nbt^2}$$

$$n = \frac{3WL}{2Pbt^2} = \frac{3 \times 3.5 \times 10^3 \times 1}{2 \times 30 \times 10^6 \times 0.09 \times (0.015)^2} = 8.64$$

$$\delta = \frac{3WL^2}{8nEt^3} = \frac{3 \times 3.5 \times 10^3 \times (1)^3}{8 \times 8.64 \times 200 \times 10^9 \times 0.09 \times (0.015)^3} = 16.669 \text{ m}$$

$$f_2 = \frac{w}{A} = \frac{4w}{7d^2} = \frac{4 \times 200}{7 \times (10)^2} = 2.55 \text{ N/mm}^2$$

$$f_0 = f_1 + f_2 = 40.76 + 2.55 = 43.31 \text{ N/mm}^2$$

ii) Axial deflection of the spring

$$s = \frac{8WD^3}{Cd^4} = \frac{8 \times 200 \times (80)^3 \times 10}{0.849 \times 10^5 \times (10)^4}$$

$$s = 9.752 \text{ mm}$$

iii) stiffness of spring $\rightarrow s = \frac{W}{s} = \frac{200}{9.752} = 20.5 \text{ N/mm}$

iv) The strain energy stored in spring \rightarrow

$$E = \frac{1}{2} ws$$

$$= \frac{1}{2} \times 200 \times 9.752$$

$$= 975.2 \text{ N-mm}$$

$$E = 0.9752 \text{ J} \quad \text{Ans}$$

Q. A leaf spring 1 m long is made up of plates each 90 mm wide and 15 mm thick. The bending stress in the spring is limited to 30 MPa. How many plates are required to enable the spring to carry a central load of 4 kN. Also calculate the deflection under the load. TAKE $E = 2 \times 10^5 \text{ MPa}$.

Sol $\rightarrow L = 1$

$$b = 90 \text{ mm} = 0.09 \text{ m}$$

$$t = 15 \text{ mm} = 0.015 \text{ m}$$

$$W = 4 \text{ kN} = 4 \times 10^3 \text{ N}$$

$$p = 30 \text{ MPa} = 30 \times 10^6 \text{ Pa}$$

$$E = 2 \times 10^5 = 2 \times 10^{11} \text{ Pa}$$